4.5 (Standard) Amplitude modulation: AM

[CH3 + Sec 4.1]

4.61. DSB-SC amplitude modulation (which is summarized in Figure 26) is easy to understand and analyze in both time and frequency domains. However, analytical simplicity is not always accompanied by an equivalent simplicity in practical implementation.

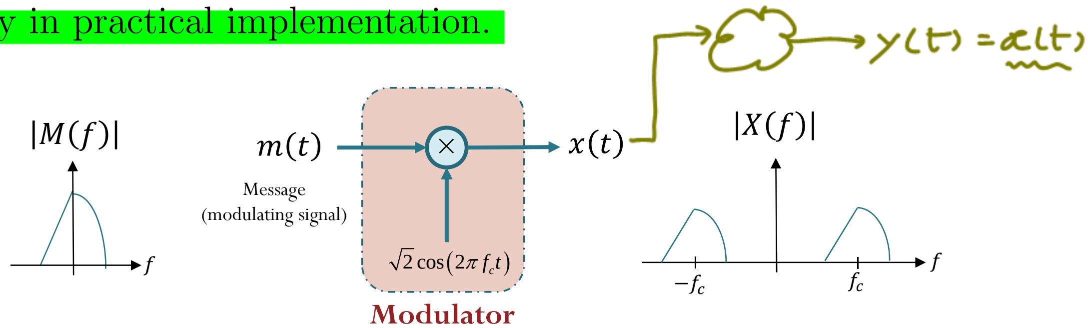


Figure 26: DSB-SC modulation.

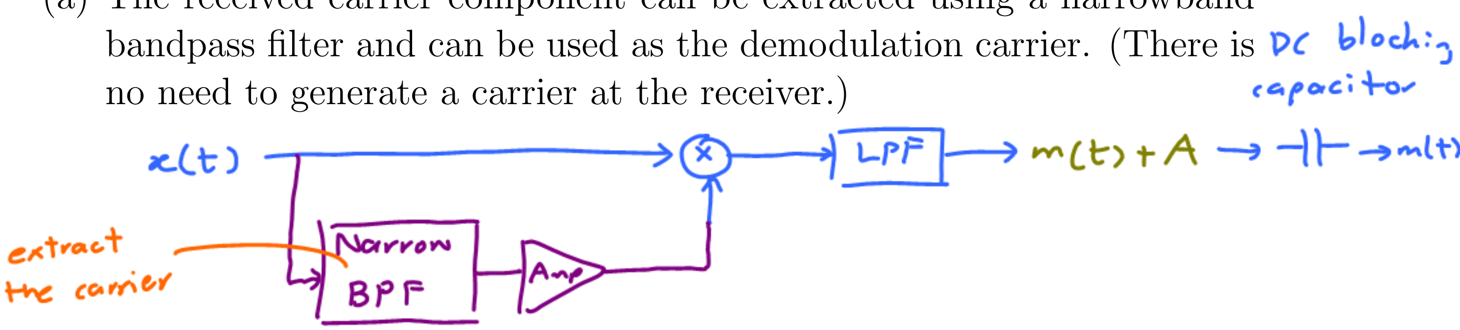
Problem: The (coherent) demodulation of DSB-SC signal requires the receiver to possess a carrier signal that is synchronized with the incoming carrier. This requirement is not easy to achieve in practice because the modulated signal may have traveled hundreds of miles and could even suffer from some unknown frequency shift.

4.62. If a carrier component is transmitted along with the DSB signal, demodulation can be simplified.

$$x(t) = m(t)\cos(2\pi f_c t) + A\cos(2\pi f_c t)$$

$$= (m(t) + A)\cos(2\pi f_c t)$$

(a) The received carrier component can be extracted using a narrowband bandpass filter and can be used as the demodulation carrier. (There is no need to generate a carrier at the receiver.)



(b) If the carrier amplitude is sufficiently large, the need to generate a demodulation carrier can be avoided completely.

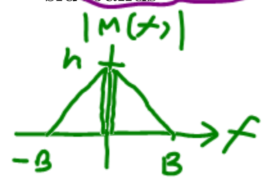
- This will be the focus of this section. ⇒ AM

$\frac{A}{2}\delta(f-f_c)+\frac{A}{2}\delta(f+f_c)$ $\quad$ $\frac{1}{2}M(f-f_c)+\frac{1}{2}M(f+f_c)$

Definition 4.63. For AM, the transmitted signal is typically defined as

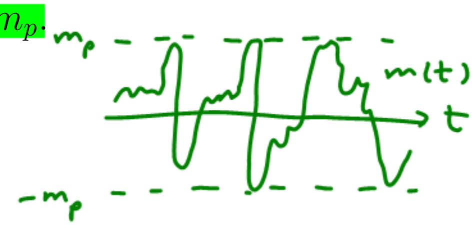
$$x_{\mathrm{AM}}(t) = \underbrace{(A+m(t))}_{A(t)}\cos(2\pi f_c t) = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t)\cos(2\pi f_c t)}_{\text{sidebands}}$$

Assumptions for $m(t)$:

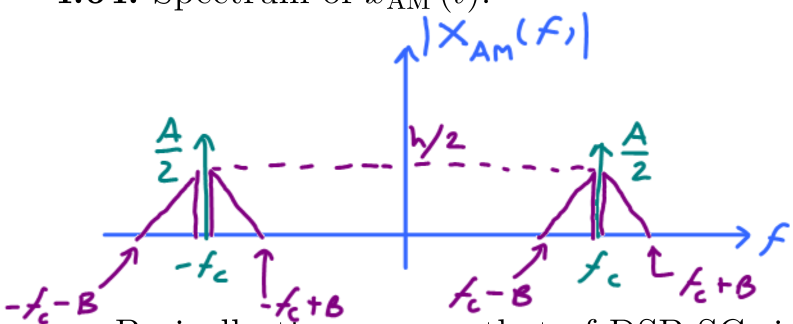


(a) Band-limited to B; that is, $|M(f)| = 0$ for $|f| > B$.

(b) Bounded between $-m_p$ and m_p; that is, $|m(t)| \le m_p$.



4.64. Spectrum of $x_{\mathrm{AM}}(t)$:



- Basically the same as that of DSB-SC signal except for the two additional impulses (**discrete** spectral component) at the carrier frequency $\pm f_c$.
  - This is why we say the DSB-SC system is a ***suppressed carrier*** system.

Definition 4.65. Consider a signal $A(t)\cos(2\pi f_c t)$. If $A(t)$ varies slowly in comparison with the sinusoidal carrier $\cos(2\pi f_c t)$, then the ***envelope*** $E(t)$ of $A(t)\cos(2\pi f_c t)$ is $|A(t)|$.

$x_{AM}(t) = (A+m(t))\cos(2\pi f_c t)$

4.66. ***Envelope of AM signal***: For AM signal, $A(t) \equiv A + m(t)$ and

$$E(t) = |A + m(t)|.$$

See Figure 27.

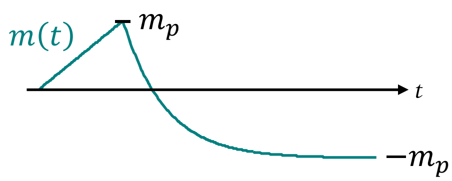
Case (a) If $\forall t$, $A(t) > 0$, then $E(t) = A(t) = A + m(t)$

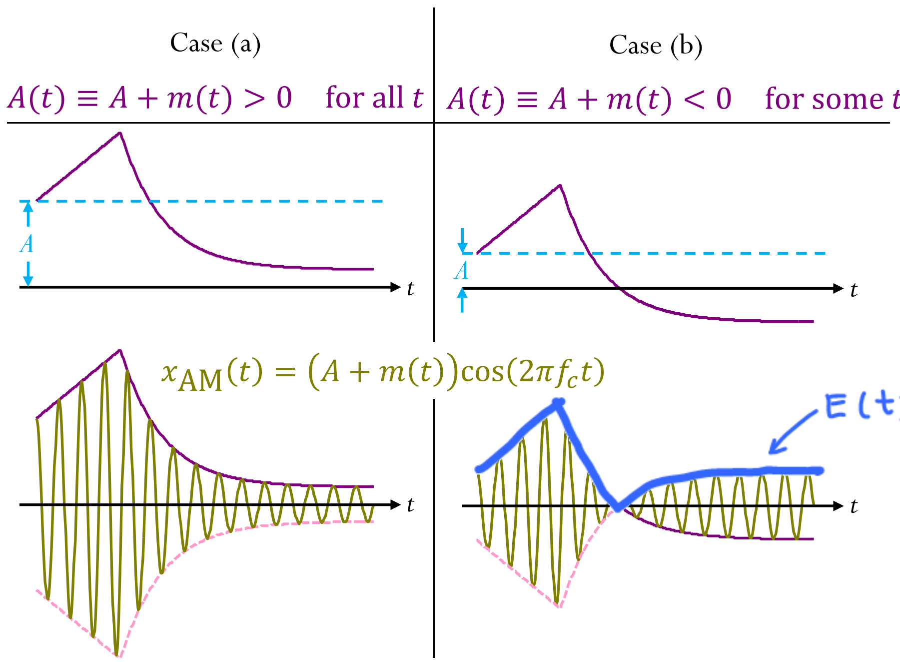
- The envelope has the same shape as $m(t)$.
- Enable envelope detection: Extract $m(t)$ from the envelope.

$A + m(t) < 0$

Case (b) If $\exists t, A(t) < 0$, then $E(t) \neq A(t)$.

- The envelope shape differs from the shape of $m(t)$ because the negative part of $A + m(t)$ is rectified.
  - This is referred to as **phase reversal** and envelope distortion.





$E(t) \neq A(t)$
$(= A + m(t))$

Figure 27: AM signal and its envelope [6, Fig 4.8]

Definition 4.67. The positive constant

$$\mu \equiv \frac{\max_t \left(\text{envelope of the sidebands}\right)}{\max_t \left(\text{envelope of the carrier}\right)} = \frac{\max_t |m(t)|}{\max_t |A|} = \frac{m_p}{A}$$

Assumption: $m_p, A > 0$

is called the **modulation index**.

- The quantity $\mu \times 100\%$ is often referred to as the **percent modulation**.

$\mu \leq 1$ $(= 100\%)$

$m_p \leq A \Rightarrow -m_p \geq -A$

$A + m(t) \geq A + (-m_p) \geq A + (-A) = 0$

$\Rightarrow$ case (a) above

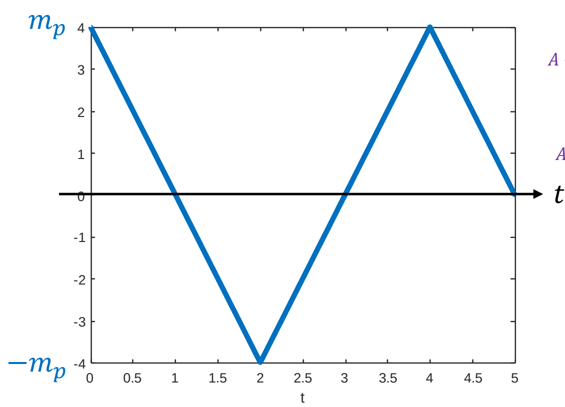
$\mu > 1$ $(= 100\%)$ $\to$ overmodulation

$A + m(t)$ may become < 0 for some t

$\Rightarrow$ case (b)

Example 4.68. Suppose $m(t)$ is plotted below. Assume that the carrier frequency f_c is large enough. Plot the corresponding AM signal $x_{\mathrm{AM}}(t)$ when the modulation index is (a) 50%, (b) 100%, and (c) 150%.

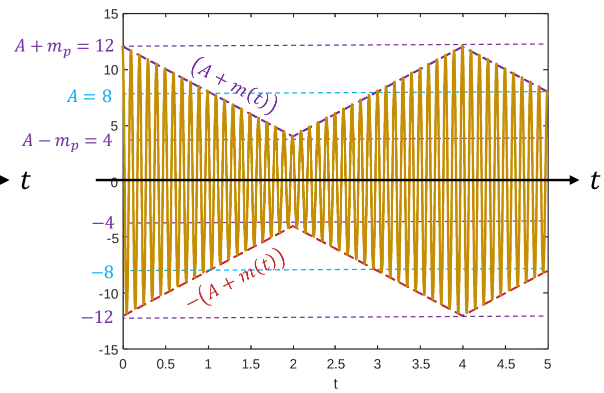
$m(t)$



$m_p = 4$

$\mu = 50\%$

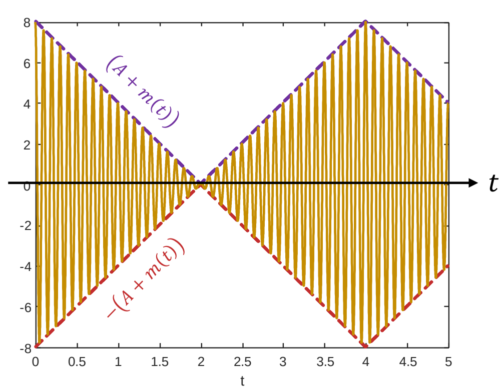
$x(t) = \left(A + m(t)\right)\cos(2\pi f_c t)$



$\mu = \frac{m_p}{A} \Rightarrow A = \frac{m_p}{\mu} = \frac{4}{0.5} = 8$

$\mu = 100\%$

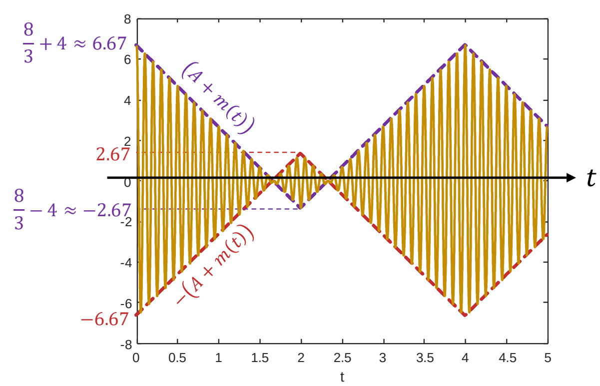
$x(t) = \left(A + m(t)\right)\cos(2\pi f_c t)$



$\mu = \frac{m_p}{A} \Rightarrow A = \frac{m_p}{\mu} = \frac{4}{1} = 4$

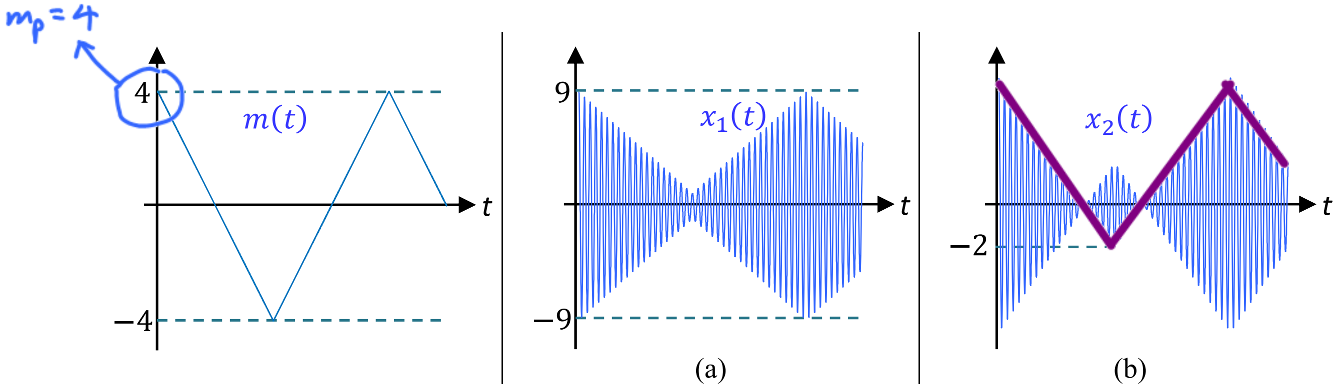
$\mu = 150\%$

$x(t) = \left(A + m(t)\right)\cos(2\pi f_c t)$

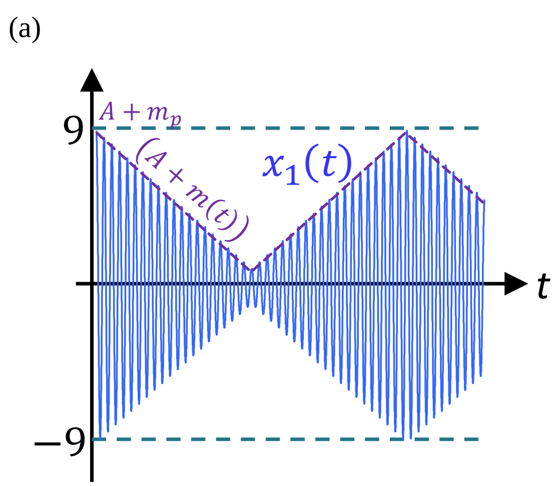


$\mu = \frac{m_p}{A} \Rightarrow A = \frac{m_p}{\mu} = \frac{4}{1.5} = \frac{8}{3}$

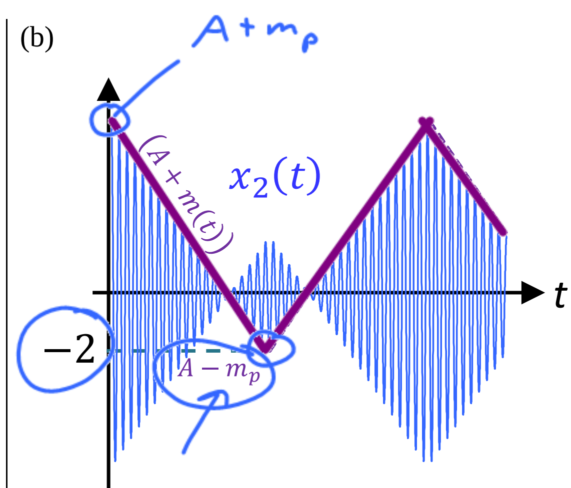
Example 4.69. Consider an AM transmission of the message $m(t)$ shown below. Two corresponding AM signals are plotted. Determine the modulation index used in each signal.



Solution:



$$A + m_p = 9$$
$$A + 4 = 9$$
$$A = 5$$
$$\mu = \frac{m_p}{A} = \frac{4}{5} = 0.8 = 80\%$$



$$A - m_p = -2$$
$$A - 4 = -2$$
$$A = 2$$
$$\mu = \frac{m_p}{A} = \frac{4}{2} = 2 = 200\%$$

Example 4.70. Consider a sinusoidal (pure-tone) message $m(t) = A_m \cos(2\pi f_m t)$. Suppose $A = 1$. Then, $\mu = A_m$. Figure 28 shows the effect of changing the modulation index on the modulated signal.

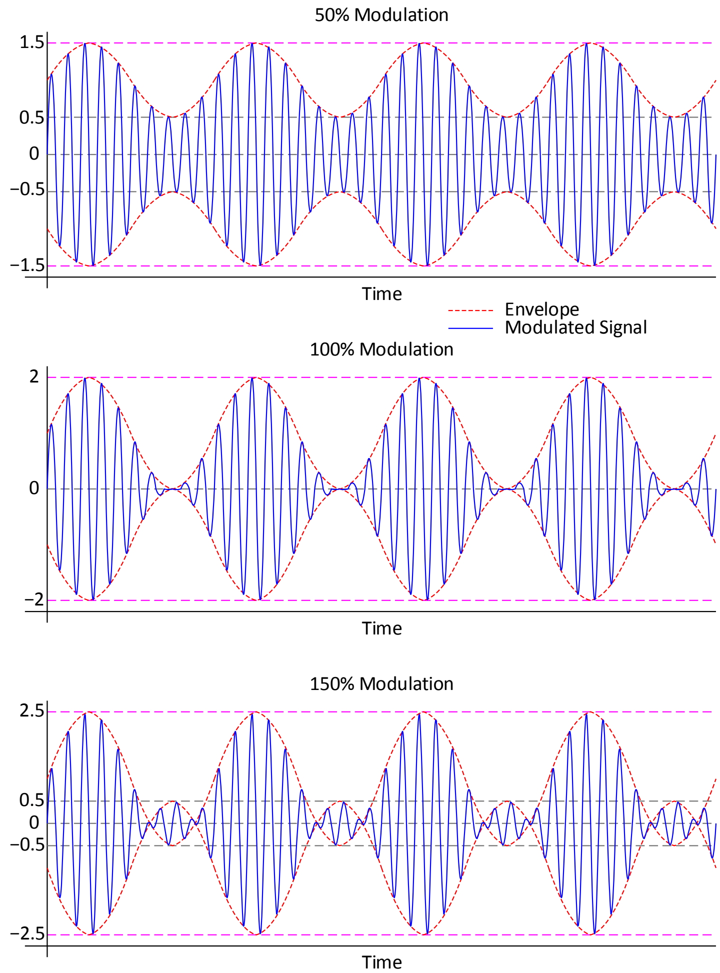


Figure 28: Modulated signal in standard AM with sinusoidal message

4.71. It should be noted that the ratio that defines the modulation index compares the maximum of the two envelopes. In other references, the notation for the AM signal may be different but the idea (and the corresponding motivation) that defines the modulation index remains the same.

- In the texbook by Carlson and Crilly, [3, p 163], it is assumed that $m(t)$ is already scaled or normalized to have a magnitude not exceeding unity ($|m(t)| \leq 1$) [3, p 163]. There,

$$x_{\mathrm{AM}}(t) = A_c(1 + \mu m(t)) \cos(2\pi f_c t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{A_c \mu m(t) \cos(2\pi f_c t)}_{\text{sidebands}}.$$

◦ $m_p = 1$

◦ The modulation index is then

$$\frac{\max_t (\text{envelope of the sidebands})}{\max_t (\text{envelope of the carrier})} = \frac{\max_t |A_c \mu m(t)|}{\max_t |A_c|} = \frac{|A_c \mu|}{|A_c|} = \mu.$$

- In [15, p 116],

$$x_{\text{AM}}(t) = A_c \left(1 + \mu \frac{m(t)}{m_p}\right) \cos(2\pi f_c t) = \underbrace{A_c \cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{A_c \mu \frac{m(t)}{m_p} \cos(2\pi f_c t)}_{\text{sidebands}}.$$

◦ The modulation index is then

$$\frac{\max_t (\text{envelope of the sidebands})}{\max_t (\text{envelope of the carrier})} = \frac{\max_t \left|A_c \mu \frac{m(t)}{m_p}\right|}{\max_t |A_c|} = \frac{|A_c| \mu \frac{m_p}{m_p}}{|A_c|} = \mu.$$

4.72. Power of the transmitted signals.

[CH3]

(a) In DSB-SC system, recall, from 4.41, that, when

$$x(t) = m(t)\cos(2\pi f_c t)$$

with f_c sufficiently large, we have

$$P_x = \frac{1}{2} P_m.$$

All transmitted power are in the sidebands which contain message information.

(b) In AM system,

$$x_{\text{AM}}(t) = \underbrace{A\cos(2\pi f_c t)}_{\text{carrier}} + \underbrace{m(t)\cos(2\pi f_c t)}_{\text{sidebands}}.$$

If we assume that the average of $m(t)$ is 0 (no DC component), then the spectrum of the sidebands $m(t)\cos(2\pi f_c t + \theta)$ and the carrier $A\cos(2\pi f_c t + \theta)$ are non-overlapping in the frequency domain. Hence, when f_c is sufficiently large

$$P_x = \frac{1}{2} A^2 + \frac{1}{2} P_m. \quad \text{useful part}$$

wasted

If $m(t)$ is sinusoidal
$m(t) = m_p \cos(2\pi f_m t + \theta)$

$P_m = \frac{1}{2} m_p^2$

$\frac{m_p^2}{P_m} = \frac{m_p^2}{\frac{1}{2}m_p^2} = 2$

$\mu = \frac{m_p}{A} \Rightarrow A = \frac{m_p}{\mu}$

- Efficiency:

useful power

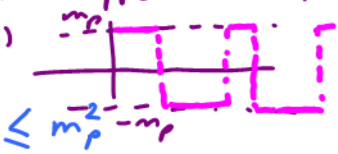
total power

$$\text{Eff.} = \frac{\frac{1}{2}P_m}{\frac{1}{2}A^2 + \frac{1}{2}P_m} = \frac{P_m}{A^2 + P_m} = \frac{1}{\frac{A^2}{P_m} + 1} = \frac{1}{1 + \frac{m_p^2}{\mu^2 P_m}} = \frac{1}{1 + \frac{2}{\mu^2}}$$

$m(t)$ is sinusoidal

- For high power efficiency, we want small $\frac{m_p^2}{\mu^2 P_m}$.
  - By definition, $|m(t)| \leq m_p$. Therefore, $\frac{m_p^2}{P_m} \geq 1$.

smallest value is 1 which happens when $m(t)$



$$\Downarrow$$

$$|m(t)|^2 \leq m_p^2 \Rightarrow P_m \equiv \langle |m(t)|^2 \rangle \leq m_p^2$$

  - Want μ to be large. However, when $\mu > 1$, we have phase reversal. So, the largest value of μ is 1.
  - The best power efficiency we can achieved is then 50%.

$$\frac{1}{1 + \frac{1}{1}\cdot 1} = \frac{1}{2} = 50\%$$

- Conclusion: at least 50% (and often close to 2/3[3, p. 176]) of the total transmitted power resides in the carrier part which is independent of $m(t)$ and thus conveys no message information.

Example 4.73. Continue from Example 4.69. Suppose $m(t)$ is a periodic triangular wave with average power $\langle m^2(t) \rangle = \frac{16}{3}$. Calculate the corresponding value of the power efficiency for each case.

Solution:

$x_{AM}(t)$	A	μ	Power Efficiency
$x_1(t)$	5	80%	$\text{Eff} = \frac{\frac{P_m}{2}}{\frac{A^2}{2} + \frac{P_m}{2}} = \frac{1}{\frac{A^2}{P_m} + 1} = \frac{1}{\frac{5^2}{16/3} + 1} = \frac{16}{91} \approx 0.1758 = 17.58\%$
$x_2(t)$	2	200%	$\text{Eff} = \frac{\frac{P_m}{2}}{\frac{A^2}{2} + \frac{P_m}{2}} = \frac{1}{\frac{A^2}{P_m} + 1} = \frac{1}{\frac{2^2}{16/3} + 1} = \frac{4}{7} \approx 0.5714 = 57.14\%$